Scale-Invariant Infinite Hierarchical Topic Model

Shusei Eshima
Department of Government, Harvard University
shuseieshima@g.harvard.edu

Daichi Mochihashi
The Institute of Statistical Mathematics
daichi@ism.ac.jp

Problem Statement

- Topic models: summarize, annotate, and categorize documents for a human reader
- Hierarchical topic models: learning hierarchical latent topic organization, especially for a large number of topics (say, > 1000) [1, 2, 3]

Existing models: fragmented tree structures

- Expected probabilities of topics decay exponentially along the depth of tree
- Heuristic rules to update topics
- Restricting the tree structure (e.g., truncating the depth to three levels)

Contributions

1. Adjusting the probability scale by considering the size of the parent topic → less fragmentation
2. Extending the tree-structured stick-breaking process (TSSB) [4] → variety of applications
3. Efficiently drawing an infinite topic tree for each document from a base infinite tree in a hierarchical Bayesian fashion

Scale-Invariant TSSB

TSSB is a crucial building block of recent neural hierarchical topic models [2, 3]. Let $\boldsymbol{\kappa} \prec \boldsymbol{\epsilon}$ indicate that $\boldsymbol{\kappa}$ is an ancestor of $\boldsymbol{\epsilon}$,

$$\pi_{\boldsymbol{\epsilon}} = \nu_{\boldsymbol{\epsilon}} \prod_{\boldsymbol{\kappa} \prec \boldsymbol{\epsilon}} (1 - \nu_{\boldsymbol{\kappa}}) \cdot \prod_{\boldsymbol{\kappa} \preceq \boldsymbol{\epsilon}} \phi_{\boldsymbol{\kappa}}, \; \phi_{\boldsymbol{\epsilon}k} = \psi_{\boldsymbol{\epsilon}k} \prod_{j=1}^{k-1} (1 - \psi_{\boldsymbol{\epsilon}j})$$

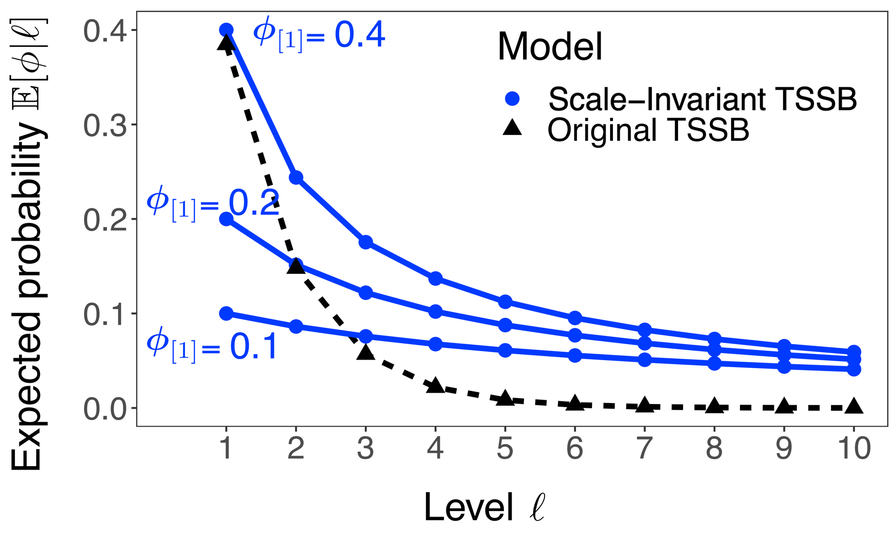
$$\nu_{\boldsymbol{\epsilon}} \sim \mathrm{Be}(1, \alpha_0), \; \psi_{\boldsymbol{\epsilon}} \sim \mathrm{Be}(1, \gamma_0).$$

- The first term: the probability of stopping at the topic $\boldsymbol{\epsilon}$ vertically.
- The next product terms: to passing ancestors of $\boldsymbol{\epsilon}$ while horizontally stopping at $\boldsymbol{\epsilon}$ and its ancestors.
- Vertical and horizontal probabilities of stopping follow Beta distribution.

The scale-invariant TSSB rescales γ_0,

$$\psi_{\boldsymbol{\epsilon}} \sim \mathrm{Be}(1, \phi_{\boldsymbol{\epsilon}'} \gamma_0).$$

Using the horizontal breaking proportion of a parent topic, $\phi_{\boldsymbol{\epsilon}'}$, to draw a *relative* stick length for its child topic → A larger break if the stick to break is shorter. The expected probabilty of horizontal break at ℓ is $\mathbb{E}[\phi \,|\, \ell] \approx 1/(2\gamma + 1/\mathbb{E}[\phi \,|\, \ell - 1])$ for $\ell \geq 2$. It was originally $\mathbb{E}[\phi \,|\, \ell] \approx 1/(2\gamma + 1)^{\ell}$.
SI-TSSB has a slower decay.



References

[1] D. M. Blei, T. L. Griffiths, M. I. Jordan, and J. B. Tenenbaum, "Hierarchical topic models and the nested Chinese restaurant process," in *NIPS*, 2003, pp. 17–24.

[2] M. Isonuma, J. Mori, D. Bollegala, and I. Sakata, "Tree-structured neural topic model," in *ACL*, 2020, pp. 800–806.

[3] Z. Chen, C. Ding, Z. Zhang, Y. Rao, and H. Xie, "Tree-structured topic modeling with nonparametric neural variational inference," in *ACL*, 2021, pp. 2343–2353.

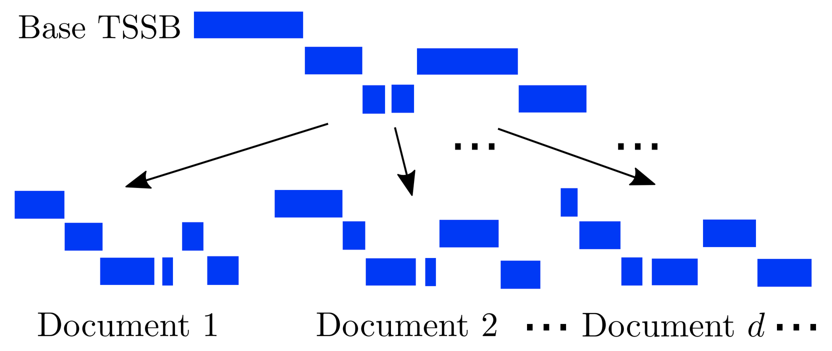
[4] R. P. Adams, Z. Ghahramani, and M. I. Jordan, "Tree-structured stick breaking for hierarchical data," in *NIPS*, 2010, pp. 19–27.

[5] Y. W. Teh, "A Bayesian interpretation of interpolated Kneser-Ney," NUS School of Computing, Tech. Rep., 2006.

Scale-Invariant Infinite Hierarchical Topic Model (ihLDA)

Document-Topic Distribution:

- We apply the hierarchical Dirichlet process separately to the vertical and horizontal probabilities.
  $\nu_{\boldsymbol{\epsilon}} \sim \mathrm{Be}(a\tau_{\tilde{\boldsymbol{\epsilon}}}, a(1 - \sum_{\boldsymbol{\kappa} \preceq \tilde{\boldsymbol{\epsilon}}} \tau_{\boldsymbol{\kappa}}))$ where $\tau_{\boldsymbol{\epsilon}} = \nu_{\boldsymbol{\epsilon}} \prod_{\boldsymbol{\kappa} \prec \boldsymbol{\epsilon}} (1 - \nu_{\boldsymbol{\kappa}})$
  $\psi_{\boldsymbol{\epsilon}k} \sim \mathrm{Be}(b\phi_{\tilde{\boldsymbol{\epsilon}}k}, b(1 - \sum_{j=1}^{k} \phi_{\tilde{\boldsymbol{\epsilon}}j}))$



Topic-Word Distribution:

- The hierarchical Pitman-Yor process [5]
- The semantic similarity between a parent topic and its children
- Increasing the specificity as the tree deepens

Inference:

- Gibbs sampling, Retrospective sampling + Binary search, Slice sampling

Data Generation Process: $\boldsymbol{\pi}^{(d)}$:= a TSSB for a doc d.

1. Draw a base TSSB $\widetilde{\boldsymbol{\pi}}$.
2. Draw topic-word distributions $H_{\boldsymbol{\epsilon}}$ from the HPY for each topic in $\widetilde{\boldsymbol{\pi}}$.
3. Draw a document-topic distribution for each document d, $\boldsymbol{\pi}^{(d)} \sim \mathrm{HTSSB}(\widetilde{\boldsymbol{\pi}})$.
4. For each word position i in a document d, draw a topic, $z_{di} \sim \boldsymbol{\pi}^{(d)}$ and draw a word, $w_{di} \sim H_{z_{di}}$.

Comparing Top Words from Three Models (max level = 3 for comparison)

Proposed:	Probabilistic: nCRP [1]	Neural: TSNTM [2]
L1: said year would also people	L1: said year one time would	L1: said show year also would
L2: said people mobile technology phone	L2: said year also would company	L2: said year game world time
L3: said software site user mail	L3: film show magic would child	L3: england first game ireland win
L2: said would government people law	L3: film indian star india actor	L3: said labour blair party election
L3: tax said government would budget	L3: film dvd effect extra man	L3: said would people law gov.
L3: labour election said party blair	L3: film harry potter dvd warner	L3: said would gov. election tax
L2: film said best award year	L2: best award film actor actress	L3: said would tax gov. election
L3: music band song year album	L2: film star story life singer	L3: said would tax gov. election
L3: game dvd film year sony	L2: film star movie actress also	L3: said would tax gov. election

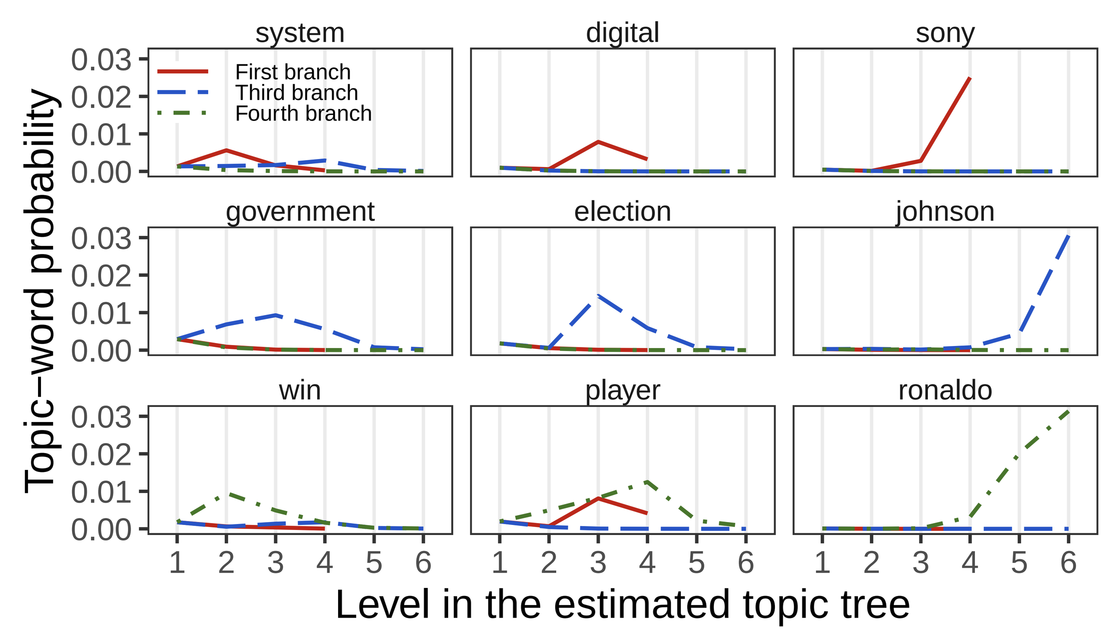
Experiments

Data:

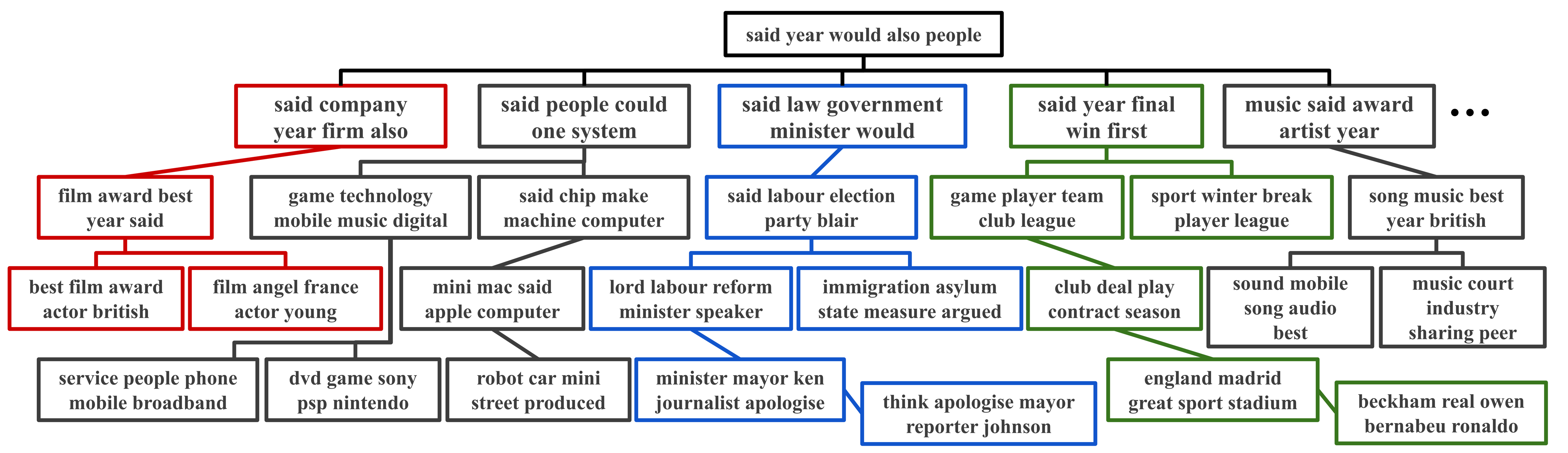
- BBC News: 2,225 documents in five topic areas from the BBC news website
- 20News: a collection of 18,828 posts from 20 USENET newsgroups
- Wikipedia: 50,153 English articles randomly sampled from ten main categories

Setup: Comparing topics with at least 100 assigned words (no truncation in parentheses)

Topic-Word Probabilities (BBC, L6)



Top Words of Selected Topics from the Estimated Topic Tree (BBC, L6)



Evaluation Metrics

Model	Max Lvl.	Tree Diversity (↑) BBC	20News	Wiki	Topic Uniqueness (↑) BBC	20News	Wiki	Average Overlap (↓) BBC	20News	Wiki	# of Topics BBC	20News	Wiki
ihLDA	3	2.24 (2.24)	**2.88** (2.86)	**2.63** (2.49)	**0.60** (0.60)	**0.82** (0.80)	**0.66** (0.63)	0.28 (0.28)	0.11 (0.14)	0.16 (0.19)	38 (38)	27 (31)	17 (18)
	≥ 4	**2.53** (2.54)	**2.88** (2.80)	2.50 (2.51)	0.55 (0.49)	0.76 (0.51)	0.65 (0.63)	0.26 (0.30)	0.12 (0.38)	0.15 (0.16)	85 (134)	67 (203)	73 (101)
nCRP	3	1.92	2.16	–	0.36	0.32	–	**0.03**	0.02	–	517	2108	–
rCRP		0.15	–	–	0.01	–	–	0.53	–	–	278	–	–
TSNTM		1.98	2.54	2.47	0.43	0.80	0.64	0.26	0.09	0.06	22	41	44
nTSNTM		2.11	2.57	2.34	0.46	0.68	0.60	0.09	**0.01**	**0.02**	68	81	111

- Tree Diversity: uniqueness of topics while considering the importance of the parent topics
- Topic Uniqueness [3]: uniqueness of all topics
- Average Overlap [3]: the average repetition rate of the top words (less overlap does not necessarily mean better interpretability)
- The results without truncation are shown in parentheses for the proposed model.